

Transportation Problem of LPP Involving Probability Density Function

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Abstract—In Linear Programming Problem (LPP), Transportation Problem (TP) is an application which is used to optimize through the probability density function of statistical approach. The main objective of this paper is to reduce complexity in Maximization problem of LPP, by fulfilling the relation between the objective function and constraints with the largest value. Here, we used non-negative integer and complex number of linear combination of form $xm\epsilon\lambda x$. It has been decided with reasonably great probability, decision region, fundamental probabilities and Laplace Transform (LT). To obtain proposed results we applied probability density function over transportation problem. According to our proposed method we implemented mathematical model through the probability density function of statistical tools. Categorically, probability density function is an approach in our proposed method to obtain the feasible solution of transportation problem, which perform better than the existing methods.

Keywords—Transportation Problem (TP), Laplace Transform (LT), Linear Programming Problem (LPP), Probability Density Function (PDF)

1 Introduction

The joint density is known of random coefficient with Linear Programming Problem (LPP) whose commutative distribution function is optimal. The optimum point of a LPP always lies on one of the corner points of the graphs feasible region. If the optimum point of LPP does not lie on either x or y axis, we can find its coordinates by drawing vertices and horizontal lines from the optimum point towards the two axis. The assumption of planner is to select the optimum one “ α ” is denoted as optimum solution of objective function in respect of operation “ β ”. In linear program vector “ t ” which is of optimum function is considered as random variable. As per the nature of problem, decide the objective function of LPP which is either of maximization or minimizations, then set the target “ T ”. The condition of maximization problem with the most probable chance should be reach with $\alpha(t) \geq T$

with the cumulative distribution. The objective is to reach up to the optimality means, some of the statistical approach like mean $E(x) = \int_0^{\infty} xf(x) dx$, $x > 0$ and variance $V(x) = E(x^2) - [E(x)]^2$ of objective function should be present with same context. In [1], [3], a single random variable under the same restrictions this problem was solved for a linear function with a coefficient of objective function or the resource vector or both. In Transportation problem of LPP, finite number of random variables are required in [2], [7], [4], [8], [9], Random costs, consumption and some application of programming to planning have been introduced, in [6],[5] the concept of stochastically and non-stochastically technological matrix concept have introduced, for all the above references are generalized for finite number of random variables for LPP. The problem which is frequently used in practice is important to obtain the information for particular types of problems. A finite number of random variables needed to decide for LPP to set its coefficients. The concept of LPP which introduced earlier by the above referred authors with their great efforts, to provide the platform to researchers of respective field are exemplary. Finally, we reach to conclude the result of given approach and proceed with some additional concept which will be more beneficial and fruitful for the author who are working over this concepts. In Section 1; Fundamental concept of probabilities and earlier result have summarized and obtained a new result which is optimum. In Section 2; the concept of Laplace Transform (LT) of finite x , $f(x)$;

$\mathcal{L} \{f(x); s\} = f^*(s) = \int_{-\infty}^{\infty} e^{-sx} f(x) dx$ have introduced to apply over the TP with the random consumption of negative exponential for optimum solution by using probability density function (PDF).

2 Mathematical Notations

In fundamental concepts and properties of Matrices and probabilities are considered to implement in introduce approach to achieve a new result, which will reduce the complexity over existing concept. Here, “A” is denoted as a matrix A and its transpose A^T . The rank of matrix is another useful factors of elementary transformation and for the solution of system of non-homogeneous linear equations. Row of a matrix is denoted as R_i and R_j , while finding the $\rho(A) \rightarrow$ Rank of A, matrix will use their elementary transformation $R_i \leftrightarrow R_j$, $R_i \leftrightarrow kR_i$, and $R_i \leftrightarrow kR_j + R_i$, to change in echelon form, $\rho(A) =$ number of non-zero rows in the echelon for (every upper triangular form is already in echelon form but every echelon form need not be a upper triangular form), $\alpha(t)$ represent as α for real valued function of $t \in T$. There are different approaches of probabilities: classical approaches (finite), empirical approaches (infinite) and Axiomatic approach, Product rule of events A and B. $P(A \cap B) = P(A \cap B) \cdot P(B/A)$; $P(A) > 0$ for compound probabilities. $P(B \cap A) = P(B) \cdot P(A/B)$, $P(B) > 0$ for conditional probabilities, when A and B are independent events, then $P(A \cap B) = P(A) \cdot P(B)$; $P(A) = (x + a)^n = \sum_{i=1}^n P(E_i) P(\frac{A}{E_i})$ Where E_1, E_2, \dots, E_n are the disjoint events $P(E_i) \neq 0$ such that “A” is arbitrary events $P(A) > 0$ which is the subset of Union $\bigcup_{i=1}^n E_i$.

We are given an m -vector, $b = (b_1, b_2, b_m)^T$, n -vector, $c = (c_1, c_2, \dots, c_n)^T$ and an $m \times n$ matrix.

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \ddots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad (1)$$

3 Terminology

3.1 Feasible Vector

Maximized or Minimized functions are called objective function. To satisfy the corresponding constraints, a vector is said to be feasible. The set of constraints are called the set of feasible vectors.

- LPP \rightarrow feasible \rightarrow constraints set is not empty.
- LPP \rightarrow Infeasible \rightarrow constraints set is empty.
- There are three possibilities for LPP.

- It may be bounded feasible.
- It may be unbounded feasible.
- It may be infeasible.

A feasible vector in which objective function achieve the value is called optimal.

3.2 LPP Converted to Standard Form

To change minimize problem to maximize problem, multiply the objective function by -1 ; similarly, if any problem constraints have negative on right side, multiply both side by -1 if the constraint.

Minimum Problem $\sum_{j=1}^n a_{ij}x_j \geq b_i$ then

Maximum problem $\sum_{j=1}^n (-a_{ij})x_j \leq -b_i$

For equality constraints: - Remove one constraint and one variable from the problem by substituting $\sum_{j=1}^n a_{ij}x_j = b_i$ for some x_j in which $a_{ij} \neq 0$.

3.3 Maximize and Minimize problem of LPP

Find an n -vector $x = (x_i)^T$,

Max $(z) = c_i^T x_i$ ($i=1, 2, n$)

$X = \{x \mid Ax \leq b, x \geq 0\}, X_i = \{x \mid Ax \leq b(t), x \geq 0\}$

Here, t is a random r -vector with probability density function $f(t_1, t_2, t_r)$ continuous and positive for $t \in T$, so, X is nonvoid and bounded.

Coefficient matrix: $A = (a_{ij})_{m \times n}$; $i=1, 2, \dots, m$; $j=1, 2, \dots, n$.

Variable vector: $x = \begin{pmatrix} x_1 \\ \cdot \\ x_n \end{pmatrix}_{nx1} = (x_i)_{nx1}; x \in X$

Constant Colum vector: $b = \begin{pmatrix} b_1 \\ \cdot \\ b_m \end{pmatrix}_{mx1} = (b_i)_{mx1}$

Here, “A” is an mxn ($m \leq n$) matrix of rank m; x and c are n-vector; b is an m-vector.

Minimize problem of LPP, Apply terminology 2(i) to change minimize to maximize.

3.4 Properties of Rank of Matrix

To find the solution of the problem of LPP by applying the properties of rank of matrix in above introduced part C. of n number of variables.

Let $\rho(A)$: Rank of $A=r$; $\rho(A \ b)$ =: Rank of augment matrix $(A \ b)=r'$

- Case I: if $r=r'=n$; solution will be unique.
- Case II: if $r=r'<n$; infinite number of solutions.
- Case III: if $r \neq r'$; inconsistent (no solution).

4 Dual General to Maximum Standard Problem

If a dual general maximum problem is transformed to standard maximum problem.

- Replace each equality constraints $\sum_j a_{ij}x_j = b_i$ by two inequality constraints $\sum_j a_{ij}x_j \leq b_i$ and $\sum_j (-a_{ij})x_j \leq -b_i$
- Replace each unrestricted variable x_j by the difference of two non-negative variables. $x_j = x_j' - x_j''$ with $x_j' \geq 0$ and $x_j'' \geq 0$.
- Repeat step ii. For minimum problem.

Let c and x are n-vectors, b and y are m-vectors, and A is an mxn matrix; we assume $m \geq 1$ and $n \geq 1$

Table 1. Primal and dual

Primal	Dual
Max $c^T x$ Subject to: $Ax \leq b$ $x \geq 0$	Min $y^T b$ Subject to: $y^T A \geq C^T$ $y \geq 0$

Table 2. Representation of dual problem

	x_1	x_2		x_n	
y_1	a_{11}	a_{12}		a_{1n}	$\leq b_1$
y_2	a_{21}	a_{22}		a_{2n}	$\leq b_2$
y_m	a_{m1}	a_{m2}		a_{mn}	$\leq b_m$
	$\geq c_1$	$\geq c_2$		$\geq c_n$	

If x and y are feasible for cTx and its dual yTb then, $cTx \leq yTb$. If a problem and its dual are feasible, then both are bounded feasible. So both are optimal for their respective problem, then equilibrium condition exist. If x^* and y^* are the feasible vectors for maximum problem and its dual respectively. Then x^* and y^* are optimal if and only if,

$$y_i^* = 0 \text{ for all } i \text{ for which } \sum_{j=1}^n (a_{ij})x_j^* < b_i \text{ and}$$

$$x_j^* = 0 \text{ for all } j \text{ for which } \sum_{i=1}^m y_i^* a_{ij} > c_j,$$

$$\text{implies } \sum_{\substack{1 \leq i \leq m \\ 1 \leq j \leq n}} y_i^* a_{ij} x_j^* = \sum_{j=1}^n c_j x_j^*$$

Such that x^* and y^* are optimal,

$$\text{Hence } \sum_{j=1}^n (c_j - \sum_{i=1}^m y_i^* a_{ij}) x_j^* = 0$$

Linear inequalities is describe $R_l(1 \leq l \leq q)$ as decision regions, x_l is a set of basic solution with base B_l ; $(r+1)$ - vectors $\delta_l = (\delta_{0l}, \delta_{1l}, \delta_{rl})$ such that $t \in R_l \Rightarrow \alpha(t) = cTx(t) = \delta_{0l} + \delta_{1l}t + \delta_{rl}t^r$

The matrix $(|\delta_k^l|, 0 \leq k \leq r, 1 \leq l \leq q)$ is a matrix of optima and p_l are the fundamental probabilities of $X = \{x \mid Ax=b, x \geq 0\}$ corresponding result for $X_t = \{x \mid Ax=b(t), x \geq 0\}$ and random variable $\alpha(t) = \min_{x \in X_t} c^T x(t)$

Let $F_l(x)$: cumulative distribution function of $\alpha(t)$.

$f_l(x)$: Probability density function of $\alpha(t)$.

$p_l(1 \leq l \leq q)$ is the fundamental probability of the stochastic LP. As we know,

The relation between cumulative distribution function and probability density function of the optimum $\alpha(t) = \alpha_l(t) (t \in S_l)$, so the function $\alpha_l(t)$ and the set S_l ; $P(S_l) = p_l$, this is valid for Linear programming of $X_t = \{x \mid Ax=b(t), x \geq 0\}$.

The random variables $\alpha_l(t)$ are non-negative and $f_l^*(s)$ is the Laplace transform of probability density function of random variable $\alpha_l(t)$, $(1 \leq l \leq q)$ and $f_\alpha^*(s)$ is the LT of probability density function of the optimum, then $f_\alpha^*(s) = \sum_{l=1}^q p_l f_l^*(s)$

5 Transportation Problem

The problem is to find numbers y_{ij} ($1 < i < m$, $1 < j < n+1$), to minimize the total transportation cost restricted by $\sum_{\substack{1 \leq i \\ 1 \leq j}} y_{ij} b_{ij} = \sum_{\substack{1 \leq i \leq m \\ 1 \leq j \leq n+1}} y_{ij} b_{ij}$.

The supply for the non-negativity constraints $y_{ij} \geq 0$ for all i and j ; $\sum_{j=1}^{n+1} (y_{ij}) \leq s_i$ for $i=1, 2, \dots, m$

The demand constraints, $\sum_{i=1}^m (y_{ij}) \geq d_j$ for $j=1, 2, \dots, n+1$, where s_i , d_j and b_{ij} are given non-negative numbers.

The transportation problem is not feasible unless supply is at least as great as demand.

$$\sum_{i=1}^m s_i \geq \sum_{j=1}^{n+1} d_j$$

The minimum cost can be defined for any $t \in T$

$$\alpha(t) = \min (y_{ij}) \sum_{\substack{1 \leq i \leq m \\ 1 \leq j \leq n+1}} y_{ij} b_{ij}, \quad (t \in T).$$

The restriction of standard transportation problem, the realization of the random variables are known before choosing the transportation plan, there are I ports, p_1, p_2, \dots, p_i that supply a certain commodity, and there are J markets, m_1, m_2, \dots, m_j , to which this commodity must be shipped. Port p_i possesses an amount s_1, \dots, s_i of the commodity ($1, 2, \dots, i$) and market m_j must receive the amount $d_1, 2, \dots, j$ let b_{ij} be the cost of transporting one unit of the commodity from port p_i to market m_j . The problem is to meet the market requirements at minimum transportation cost. So, it is assumed that we cannot send a negative amount from p_i to m_j , we have $y_{ij} \geq 0$ for i, j , under the assumption, simple algorithm is required to make the total demand is equal to total supply. To make the assumption positive by the device of adding to the problem a dummy market, i.e. dump, which fulfil the requirement to make total demand equal to total supply, such that transportation costs to the dump are zero.

$$\sum_{i=1}^m s_i = \sum_{j=1}^{n+1} d_j$$

So, it is sufficient to find feasible vectors, y_{ij} , u_i and v_j , that satisfy the equilibrium condition which reduce for all i and j ; $y_{ij} > 0$ implies that $v_j - u_i = b_{ij}$; now, this problem can be easily solve by simplex method, the simplex tableau for this problem involves an ij by $i+j$ constraint matrix. For this types of problem, it is easy to implement transportation problem instead of simplex method, because it may describe easily in terms of i and j transportation array.

Table 3. Representation of Transportation array

	m1	m2		mj	
p1	b11 y11	b12 y12		b1j y1j	s1
p2	b21 y21	b22 y22		b2j y2j	s2
pi	bi1 yi1	bi2 yi2		bij yij	si
	d1	d2		dj	

The algorithm, which consists three parts:

- Find a basic feasible shipping schedule y_{ij} .
- Test for optimality
- If the test fails, find an improved basic feasible shipping schedule, and above.

The main aim of transportation problem is to find the distribution of minimum total costs with its mean value and variance, by applying probability density function over number α_j which is positive.

$$\phi_j(x) = \{\lambda_j e^{-\alpha_j x} \text{ if } x \geq 0 \text{ and } 0 \text{ if } x < 0 \text{ for } 1 \leq j \leq n\};$$

As we know, if l_i ($1 \leq i \leq d$) are independent non-negative random variables with probability density function

$$\phi_i(x), 0 \leq x \leq d.$$

$f_i^*(s)$ is the LT of the probability density function of random variable $\alpha_i(t)$ where $1 \leq i \leq q$ and $f_\alpha^*(s)$ is the LT of probability density function of optimum:

$$f_\alpha^*(s) = \sum_{i=1}^q p_i f_i^*(s).$$

By applying the above theorem, we can obtain

$$\mathcal{L}(\alpha, e^{-\alpha \cdot x}, s) = \int_0^\infty \alpha e^{-\alpha x} e^{-sx} dx = \alpha / (\alpha + s);$$

By applying elementary properties of LT of

$$f_\alpha(x) = \sum_{1 \leq i \leq q} \alpha_{ij} A_{ij} \psi_{ij}(x); \text{ Where, } \psi_{ij}(x) = e^{-\alpha_{ij}(x - \lambda_0^i)} \text{ if } x \geq \lambda_0^i, \text{ otherwise } 0.$$

λ_r^i is $(r+1)$ vectors, $t = (t_1, t_2, t_r)$ a random vector with value in R_r , Problem T is an r -dimensional interval.

We obtained probability density function of optimum

$$F_\alpha(x) = \sum_{1 \leq i \leq q} A_{ij} (1 - B_{ij} e^{-\alpha_{ij} x}) \text{ where } B_{ij} = e^{-\lambda_0^i \alpha_{ij}} \text{ \& } F(\infty) = 1,$$

The obtained results are useful to implement over transportation problem to conclude our approach.

6 Conclusion

The concept of this paper, to use the non-negative integer and complex number of linear combination of form $xm\epsilon\lambda x$ which is a probability density function of minimum total transportation costs. These are used and implemented in our proposed

approach to reduce the complexity and improve the accuracy of result with the distribution of continuous non-negative random variables. For this, the LT of the probability density function is a rational function of 's' having arbitrary mean and any coefficient variation between 1 and $+\infty$. The introduced approach can be used to solve different Linear Programming Problems to obtain probability density function of optimum which perform better than the existing methods that is frequently refer to a transportation problems.

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